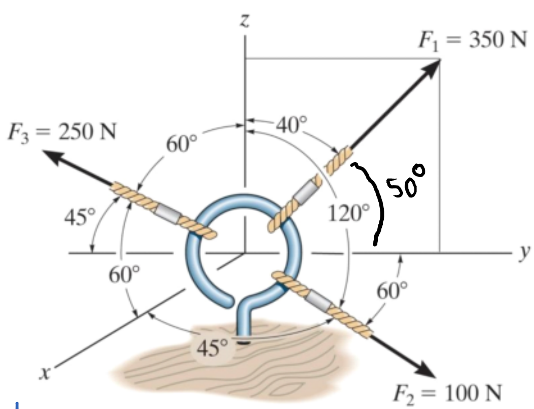
Example



Note that the angles do not all lie in the same plane!

The cables attached to the screw eye are subjected to the three forces shown.

(a) Express each force vector using the Cartesian vector form (components form).

(b) Determine the magnitude of the resultant force vector

(c) Determine the direction cosines of the resultant force vector

a) Start with $\underset{\sim}{F}_1$

$$\underset{\sim}{F}_1 = 0 \cdot \hat{i} + F_{1y}\hat{j} + F_{1z} \cdot \hat{k}$$

$$= F_1\left(0 \cdot \hat{i} + \underbrace{\frac{F_{1y}}{F_1}}_{\cos(50^\circ)}\hat{j} + \underbrace{\frac{F_{1z}}{F_1}}_{\cos(40^\circ)}\hat{k}\right)$$

$$= F_1\left(\cos 50^\circ \hat{j} + \cos 40^\circ \hat{k}\right)$$

$$= (350\,N)\left(\cos 50^\circ \hat{j} + \cos 40^\circ \hat{k}\right)$$

$$= \left(225\,\hat{j} + 268\,\hat{k}\right) N$$

$$\underset{\sim}{F}_2 = F_2 \cdot \left(\cos 45^\circ \hat{i} + \cos 60^\circ \hat{j} + \cos 120^\circ \hat{k}\right)$$

$$= (100N)\left(\frac{\sqrt{2}}{2}\hat{i} + \frac{1}{2}\hat{j} - \frac{1}{2}\hat{k}\right)$$

$$= \left(70.7\,\hat{i} + 50\,\hat{j} - 50\,\hat{k}\right) N$$

$$\underset{\sim}{F}_3 = F_3 \cdot \left(\cos 60^\circ \hat{i} + \cos 135^\circ \hat{j} + \cos 60^\circ \hat{k}\right)$$

$$= \left(125\,\hat{i} - 177\,\hat{j} + 125\,\hat{k}\right) N$$

b) Resultant force: $\Sigma \underset{\sim}{F} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3$

in $\hat{i}$ direction: $\Sigma \underset{\sim}{F}_x = (0 + 70.7 + 125)N \cdot \hat{i} = 196 N \hat{i}$

in $\hat{j}$ direction: $\Sigma \underset{\sim}{F}_y = (225 + 50 - 177)N \cdot \hat{j}$

$= 98 N \hat{j}$

in $\hat{k}$ direction: $\Sigma \underset{\sim}{F}_z = (268 - 50 + 125) N \hat{k}$

$= 343 N \hat{k}$

$$\Sigma \underset{\sim}{F} = (196 \hat{i} + 98 \hat{j} + 343 \hat{k}) N$$

$$|\Sigma \underset{\sim}{F}| = \sqrt{(196)^2 + (98^2) + (343)^2}\ N = 407 N$$

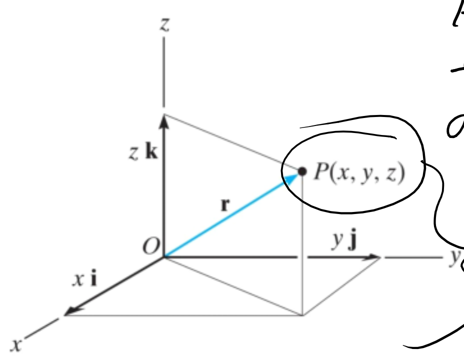
c) Direction cosines of $\Sigma \underset{\sim}{F}$:

$$\frac{\Sigma \underset{\sim}{F}}{|\Sigma \underset{\sim}{F}|} = \frac{196}{407} \hat{i} + \frac{98}{407} \hat{j} + \frac{343}{407} \hat{k}$$

$$= 0.481 \hat{i} + 0.241 \hat{j} + 0.843 \hat{k}$$

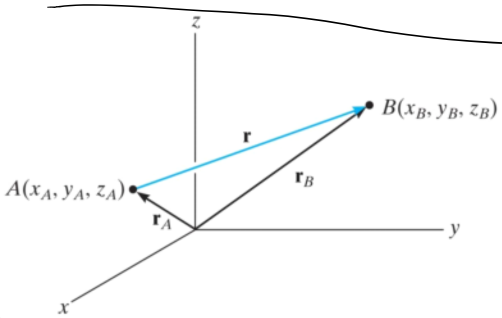
Position vectors

A position vector $\underset{\sim}{r}$ is a fixed vector that locates a point in space relative to another point



here, $\underset{\sim}{r} = x\hat{i} + y\cdot\hat{j} + z\cdot\hat{k}$ give point $P(x,y,z)$ relative to the origin.

(might write as $\underset{\sim}{r} = \underset{\sim}{r}_{OP}$)



$\underset{\sim}{r}$ is position of B relative to A

$$\underset{\sim}{r}_A + \underset{\sim}{r} = \underset{\sim}{r}_B$$

$$\Rightarrow \underset{\sim}{r} = \underset{\sim}{r}_B - \underset{\sim}{r}_A$$

$$= (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$

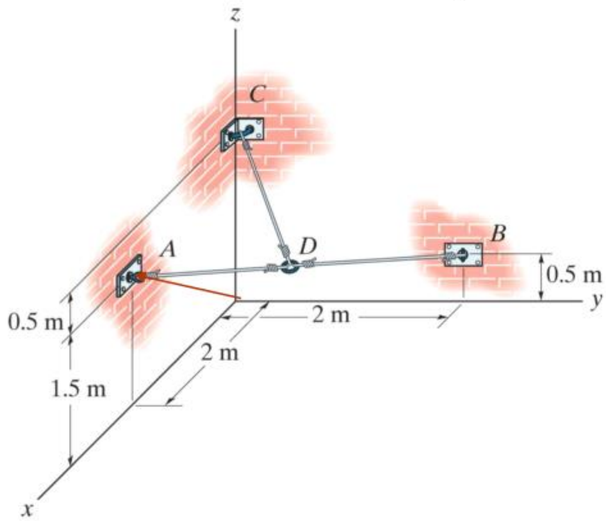
(might write $\underset{\sim}{r}$ as $\underset{\sim}{r}_{AB}$)

$$\underset{\sim}{r}_{AB} = \underset{\sim}{r}_B - \underset{\sim}{r}_A$$

$$= \underset{\sim}{r}_{OB} - \underset{\sim}{r}_{OA}$$

Example

The ring at D is midway between points ***A*** and ***B***. Determine the lengths of wires AD, BD and CD



D is midway between A & B:

$$x_D = \frac{x_A + x_B}{2} = \frac{(2+0)\,m}{2} = 1\,m$$

$$y_D = \frac{y_A + y_B}{2} = \frac{(0+2)\,m}{2} = 1\,m$$

$$z_D = \frac{z_A + z_B}{2} = \frac{(1.5+0.5)\,m}{2} = 1\,m$$

$$\Rightarrow \underset{\sim}{r}_{OD} = (1\,m)(\hat{i} + \hat{j} + \hat{k})$$

$$\underset{\sim}{r}_{OA} = (2\hat{i} + 0\cdot\hat{j} + 1.5\hat{k})\,m$$

$$\underset{\sim}{r}_{OA} + \underset{\sim}{r}_{AD} = \underset{\sim}{r}_{OD} \Rightarrow \underset{\sim}{r}_{AD} = \underset{\sim}{r}_{OD} - \underset{\sim}{r}_{OA} = \left[(1-2)\hat{i} + (1-0)\hat{j} + (1-1.5)\hat{k}\right]m$$

$$= \left[-1\cdot\hat{i} + 1\cdot\hat{j} - 0.5\cdot\hat{k}\right]m$$

$$\underset{\sim}{r}_{BD} = \underset{\sim}{r}_{OD} - \underset{\sim}{r}_{OB} = \left[(1-0)\hat{i} + (1-2)\hat{j} + (1-0.5)\hat{k}\right]m$$

$$= (1\cdot\hat{i} - 1\cdot\hat{j} + 0.5\cdot\hat{k})\,m$$

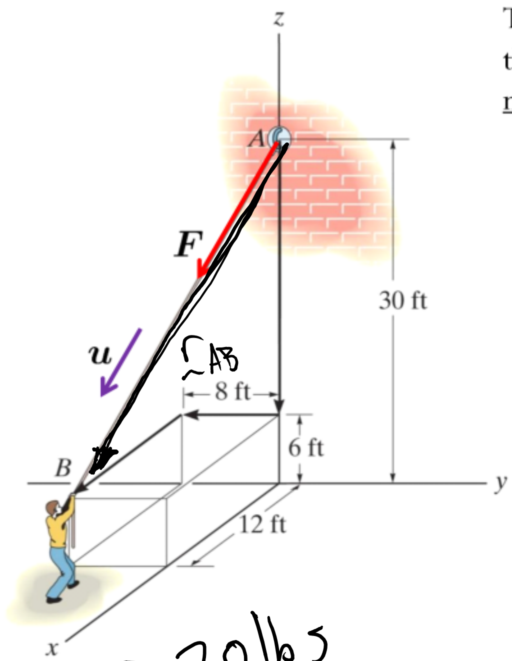
$$\underset{\sim}{r}_{CD} = \underset{\sim}{r}_{OD} - \underset{\sim}{r}_{OC} = \left[(1-0)\hat{i} + (1-0)\cdot\hat{j} + (1-2)\hat{k}\right]m$$

$$= (1\cdot\hat{i} + 1\cdot\hat{j} - 1\cdot\hat{k})\,m$$

Cable lengths are the magnitudes of the position vectors:

$$\boxed{\begin{aligned} r_{AD} &= \sqrt{(-1m)^2 + (1m)^2 + (-0.5m)^2} = 1.50m \\ r_{BD} &= \sqrt{(1m)^2 + (-1m)^2 + (0.5m)^2} = 1.50m \\ r_{CD} &= \sqrt{(1m)^2 + (1m)^2 + (-1m)^2}\,m = 1.73m \end{aligned}}$$

Force vector directed along a line



The force vector $\boldsymbol{F}$ acting a long the rope can be defined by the unit vector $\boldsymbol{u}$ (defined the direction of the rope) and the magnitude of the force.

$$\underset{\sim}{F} = F \cdot \hat{u}$$

vector magnitude

unit vector in the direction of the rope

$$\hat{u} = \frac{\underset{\sim}{r}}{|\underset{\sim}{r}|} = \frac{\underset{\sim}{r}}{r}$$

$F = 70\,\text{lbs}$

$$\underset{\sim}{r}_{AB} = (\underset{\sim}{r}_B - \underset{\sim}{r}_A) = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$

$$= (12\hat{i} - 8\hat{j} - 24\hat{k})\,ft$$

$$\text{length} = \sqrt{(12\,ft)^2 + (-8\,ft)^2 + (-24\,ft)^2}$$

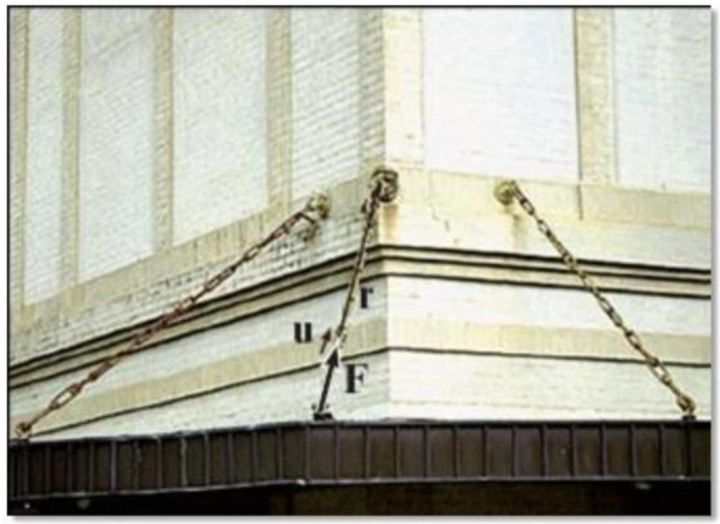
$$= 28\,ft$$

$$\hat{u} = \frac{12}{28}\hat{i} - \frac{8}{28}\hat{j} - \frac{24}{28}\hat{k}$$

$$F = F \cdot \hat{u} = (70\,lbs)\,\hat{u}$$

$$= (30\hat{i} - 20\hat{j} - 60\hat{k})\,\text{lbs}$$

Force vector directed along a line

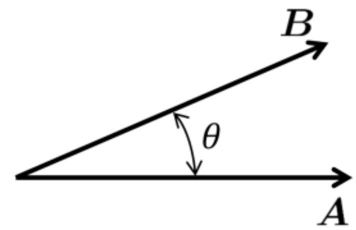


Don't look up!

Dot (or scalar) product

The dot product of vectors **A** and **B** is defined as such

$$\boxed{\boldsymbol{A}\bullet\boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}|\,\cos(\theta)}$$



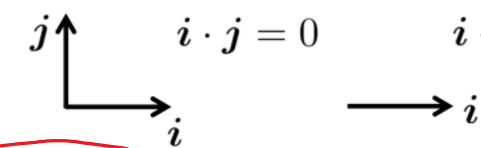
Laws of operation:

$$\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{B}\cdot\boldsymbol{A}$$

$$\alpha(\boldsymbol{A}\cdot\boldsymbol{B}) = \alpha\boldsymbol{A}\cdot\boldsymbol{B} = \boldsymbol{A}\cdot\alpha\boldsymbol{B}$$

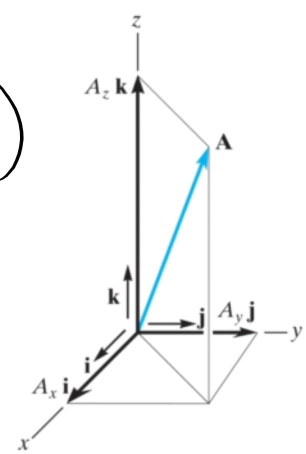
$$\boldsymbol{A}\cdot(\boldsymbol{B}+\boldsymbol{C}) = \boldsymbol{A}\cdot\boldsymbol{B} + \boldsymbol{A}\cdot\boldsymbol{C}$$

Note that:



$\boldsymbol{i}\cdot\boldsymbol{j} = 0$ $\boldsymbol{i}\cdot\boldsymbol{i} = 1$

Cartesian vector formulation:



$$\boxed{\underset{\sim}{A}\bullet\underset{\sim}{B}} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\bullet(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x\cancelto{1}{\hat{i}\bullet\hat{i}} + A_xB_y\cancelto{0}{\hat{i}\bullet\hat{j}} + A_xB_z\cancelto{0}{\hat{i}\bullet\hat{k}}$$

$$+ A_yB_x\cancelto{0}{\hat{i}\bullet\hat{j}} + A_yB_y\cancelto{1}{\hat{j}\bullet\hat{j}} + A_y\cdot B_z\cancelto{0}{\hat{j}\bullet\hat{k}}$$

$$+ A_zB_x\cancelto{0}{\hat{i}\bullet\hat{k}} + A_zB_y\cancelto{0}{\hat{j}\bullet\hat{k}} + A_z\cdot B_z\cancelto{1}{\hat{k}\bullet\hat{k}}$$

$$= \boxed{A_xB_x + A_yB_y + A_zB_z}$$

$$= \boxed{A_x B_x + A_y B_y + A_z B_z}$$